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Optimal Dynamic Environmental Policies of a Profit Maximizing Firm

By

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In this paper we study the optimal environmental policy of the firm for different scenarios dependent on (costs of) production technologies, financing costs, and governmental policy. The governmental instruments to be considered are:
- investment grants on cleaner production technologies and on abatement activities;
- taxes imposed on environmental pollution.

The problem is defined as an optimal control model. In this model, the firm influences its pollution output through the choice of its production technology. Available are a more capital-extensive and dirty activity, a more capital-intensive and clean activity, and an abatement activity that eliminates pollution completely or partially.

1. Introduction

The aim of our paper is to establish conditions under which the firm is willing to diminish pollution by either using a less pollution-intensive production technology or by investing in an "end of pipe" or abatement technology. How can the government stimulate the firm (by using which instruments) to such a change of technology?

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From an economic point of view, environment has thus become a scarce commodity. Consequently, environmental use is an allocation problem (Siebert, 1987) and this viewpoint lies at the basis of several publications in environmental economics (e.g., Baumol and Oates, 1988; Bohm and Kneese, 1971). At the macroeconomic level a lot of attention has been paid to the analysis of the trade-off relation between economic growth and environmental quality (e.g., Forster, 1977; Luptácik and Schubert, 1982), while the analysis of the effects of governmental regulation on the firm's decision-making concerning pollution control spending, employment, and investment is an important issue at the microeconomic level (e.g., Feichtinger and Luptácik, 1987; and Lin, 1987).

In these studies several governmental instruments are mentioned through which the government tries to influence corporate policy, which is one of the topics of this paper. One class of instruments includes direct controls by setting limits to the amounts of effluent that the factories can discharge into a stream (environmental standards, see, e.g., Beavis and Dobbs, 1986). Another mechanism for the attainment of a given environmental target is the standard-price-approach introduced by Baumol and Oates (1971). The basic idea of this concept is to meet a given quantity of emissions by rationing the demand for emission permits by prices.

The relative efficiency of standard setting and emissions taxation in connection with pollution control is analyzed by Baron (1985). The paper by Magat (1978) extends the static comparison of effluent taxes and efficient standards to a dynamic world in which firms invest resources in improving their abatement technology as well as their production technology.

As stated at the beginning of the section, in this paper we study corporate policy from the management's point of view. Management has to reckon with a government using instruments like emissions tax and investment grants in order to encourage the use of cleaner production technologies and/or abatement activities. Under these conditions the policy of the firm consists of decisions about the level of production and the choice of production activities, which in our model not only fix the sales value and production costs, but also the level of pollution.

We assume that management maximizes the shareholders' value of the firm. We present the resulting optimal policies of the firm under different scenarios. Each scenario is characterized by a different set of values for: factor productivities, unit costs, price/demand curve, financing costs, restriction on the capital structure, governmental instruments on pollution, and profit tax rate.

The problem will be analyzed by developing a dynamic model of the firm, which is an extended version of the one described by van Loon (1983) (see also Hartl, 1988). In section 2 important assumptions are presented, and in section 3 the dynamic model of the firm is introduced. In section 4 we present the results and analyze the optimal policies for two different scenarios emphasizing economic aspects. Also, we study the differences between the emissions tax and the investment grant as far as their impacts on the firm's optimal policy is concerned. Section 5 concludes the paper and the technical analysis can be found in the appendices.

2. Preliminaries

In this section we present important assumptions of the model. These assumptions concern the environmental policy of the government, the way the firm can influence its own environmental pollution output, the firm's possibilities to finance its activities, and the goal the firm wants to reach in taking its decisions.

As mentioned in the introduction we incorporate the following governmental environmental instruments in our model:

- investment grants on cleaner production technologies and on abatement activities;
- taxes imposed on environmental pollution.

The level of the firm's pollution depends on, beside the level of output, the choice of the production technology. For simplicity, we assume that the firm produces only one homogeneous product. We also assume that at the start of the planning period the firm uses a more capital-extensive and dirty technology. The firm may consider to replace this technology by a more capital-intensive and cleaner one (see, e.g., Kistner, 1983) and/or to build an abatement installation that eliminates pollution completely or partially. This choice will depend on the unit costs of the different combinations of technologies. The amount of time the firm needs to reach its ideal combination is determined by the level of available cash flow (from operations and from funding activities). Funding activities consist of attracting debt money. In this model we assume that there is a maximum debt to equity rate.

The level of internal generated cash flow depends on the return on the invested capital and on the level of corporate profit taxation. We further assume that shareholders do not demand dividend each year, provided that management maximizes the value of the firm. This value of the firm consists of the discounted dividend payments over the planning period and the discounted value of the firm at the horizon date.

The financial situation described in this model holds for many firms: equity can only be increased by retained earnings and subsidies, which are apart from profits, and the total capacity of debt financing depends heavily on the (book) value of equity, thus on the value of total assets.

3. The Model

The firm is able to produce a homogeneous output through two different techniques, a capital-extensive activity and a capital-intensive one:

$$Q(K_1, K_2) = q_1 K_1 + q_2 K_2 \ , \tag{1}$$

$$q_1 > q_2 \ , \tag{2}$$

in which:
K_j amount of capital goods assigned to activity j,
Q production rate,
q_j capital productivity of activity j.

Production through these two activities causes environmental pollution, where activity 2 is relatively more clean than activity 1. The latter assumption is imposed from the efficiency point of view. If activity 2 is the more dirty one, then it will never be used by the firm because it also holds that the capital productivity of activity 2 is lower than that of activity 1. Besides, it is also possible for the firm to invest in a technique that cleans pollution. We assume that pollution is homogeneous by nature:

$$E(K_1, K_2, K_3) = e_1 K_1 + e_2 K_2 - e_3 K_3 \ , \tag{3}$$

$$\frac{e_1}{q_1} > \frac{e_2}{q_2} \ , \tag{4}$$

in which:
E amount of emissions,
K_3 amount of capital goods assigned to the abatement activity 3,
e_j emission to capital rate of activity j; $j = 1, 2$,
e_3 abatement to capital rate of activity 3.

There are no unused capital goods, so all capital goods are assigned to any of the three activities:

$$K = K_1 + K_2 + K_3 \ , \tag{5}$$

in which:
K capital goods stock.

Because the labor to capital rate differs among activities, the firm's policy also influences the level of employment:

$$L(K_1, K_2, K_3) = l_1 K_1 + l_2 K_2 + l_3 K_3 \ , \tag{6}$$

$$l_1 \neq l_2 \neq l_3 \ , \tag{7}$$

in which:
L level of employment of the firm,
l_j labor to capital rate of activity j.

The stock of finished products is constant over time, which implies that at each point of time the level of production equals the level of sales. If the firm raises output, its (net) selling price will decrease:

$$S(Q) = P(Q)Q \ , \tag{8}$$

$$S'(Q) > 0; \quad S''(Q) < O; \quad S(0) = 0 \ , \tag{9}$$

in which:
S sales rate,
P (net) selling price.

In this model the only asset is capital goods which can be financed by equity and debt. The value per unit of a capital good is fixed at one unit of money. So the balance sheet equation is:

$$X + Y = K_1 + K_2 + K_3 \ , \tag{10}$$

in which:
X stock of equity,
Y stock of debt.

The firm can raise its equity by retained earnings and by acquiring investment grants. However, equity is reduced through the tax imposed on environmental pollution (which cannot be deducted from profit before taxes):

$$\dot{X} = R + g(I_2 + I_3) - f_2 E \ , \tag{11}$$

in which:
I_j investment rate assigned to activity j,
R retained earnings rate,
f_2 emission tax rate,
g investment grant rate on the cleaner activity 2 and on the abatement activity 3.

To construct the profit and loss account-equation we introduce the following assumptions:

- corporate tax is proportional to profit;
- depreciation is proportional to the stock of capital goods;
- borrowing does not carry any transaction costs.

Then the flow of retained earnings can be formulated as:

$$R = (1 - f_1)[S - w\,L - a\,K - r\,Y] - D\ , \tag{12}$$

in which
D dividend rate,
a depreciation rate,
f_1 corporate profit tax rate,
r interest rate on debt,
w wage rate.

For each technology the usual formula of net investment applies:

$$\dot{K}_j = I_j - a\,K_j; \quad j = 1, 2, 3\ . \tag{13}$$

The amount of debt is bounded from above (see van Loon, 1983), i.e.:

$$Y \leq k\,K \iff Y \leq \frac{k}{1-k} X\ , \tag{14}$$

$$0 \leq k < 1\ , \tag{15}$$

in which:
k maximum debt to capital rate.

Assume that the firm behaves so as to maximize the shareholders' value of the firm. This value consists of the sum of the discounted dividend stream over the planning period and the discounted final value of the firm at the end of the planning period. This final value is equal to the difference between the final value of the assets and the sum of the final stock of debt and the amount of investment grants that have to be repaid due to stopping corporate activity:

$$\begin{aligned} \underset{D, I_1, I_2, I_3}{\text{maximize}} & \int_0^z e^{-i\,T} D(T)\,\mathrm{d}T \\ & + e^{-iz}\Big[K(z) - Y(z) - g(K_2(z) + K_3(z))\Big]\ , \end{aligned} \tag{16}$$

in which:

T time, $0 \leq T \leq z$,

i shareholders' time preference rate,

z horizon date.

To complete the model, we add some non-negativity conditions:

$$D \geq 0, \quad Y \geq 0, \quad X \geq 0, \quad E \geq 0, \\ K_1 \geq 0, \quad K_2 \geq 0, \quad K_3 \geq 0\,, \tag{17}$$

$$X(0) = X_0 > 0, \quad K(0) = K_0 > 0\,. \tag{18}$$

The controls D and I_j $(j = 1, 2, 3)$ do not need to be explicitly bounded from above, because they have an implicit upper bound induced by the model's financial structure.

As we will show later on, it is convenient to distinguish between different cases, depending on the mode of production, the financial structure, and the dividend pay out rate. For each case, we denote the resulting unit cost by:

$$c_{bn}; \qquad b \in \{1, 2, 12, 13, 23, 123\}\,, \\ n \in \{X, Y, YX, XD, YD\}\,, \tag{19}$$

in which:

b activity performed by the firm (e.g., $b = 123$ means that the three activities are performed together),

n index of financial structure and dividend pay out rate:

- $n = X$: self-financing case,
- $n = Y$: maximum debt financing case,
- $n = YX$: intermediate debt financing case,
- $n = XD$: self-financing case together with a positive dividend pay out rate,
- $n = YD$: maximum debt financing case together with a positive dividend pay out rate.

The firm never performs only activity 3 because of its non-productivity. Due to the following assumption it is not optimal to pay out dividend in the intermediate debt financing case:

$$i \neq (1 - f_1)r\,. \tag{20}$$

This assumption indicates that the capital market is imperfect (see also van Loon, 1983).

To limit the number of possible solutions we assume that under all circumstances the output per unit equity of activity 1 is larger than the output per unit equity of activity 2 (notice that activity 3 is non-productive). In Appendix 1 a set of inequalities is presented that ensure the validity of this assumption.

We further assume that at the start of the planning period the firm wants to grow as fast as possible. Due to the previous assumption we can conclude that this can be achieved by using the capital-extensive and dirty activity 1:

$$K(0) = K_1(0) \; . \tag{21}$$

To make sure that this initial firm behavior is optimal we assume a sufficiently small initial level of the capital goods stock so that the marginal sales level exceeds each of the unit costs:

$$\begin{aligned} S'(Q(0)) > \max c_{bn}; \qquad & b \in \{1, 2, 12, 13, 23, 123\} \; , \\ & n \in \{X, Y, YX, XD, YD\} \; . \end{aligned} \tag{22}$$

We exclude solutions that are not well defined by assuming:

$$\begin{aligned} c_{bn} \neq c_{jm}; \qquad & b, j \in \{1, 2, 12, 13, 23, 123\}, \\ n, m \in \{X, Y, YX, XD, YD\}, & \quad bn \neq jm \; . \end{aligned} \tag{23}$$

In Appendix 2 we show that the model can be simplified into a model that contains 2 state variables, 4 control variables, and 9 restrictions. In Appendix 3 we present the necessary conditions for an optimal solution, which are derived by using Pontryagin's maximum principle. We also explain in what way these conditions are transformed into the optimal trajectories of the firm.

4. The Firm's Optimal Trajectories

The optimal policy of the firm depends on the scenario in which it has to operate. From the optimal solution, 16 different scenarios can be discerned, each asking for a different optimal policy of the firm. Such a policy causes an expansion of the firm during which growth and consolidation are alternating stages. If the planning horizon is far enough, these 16 policies lead to 8 different final stages. Which of these final stages is the optimal one depends on 3 characteristics of the scenario: financial costs, technology, and environmental policy of the government.

Financial Costs

Main issue here is whether cost of equity is larger than cost of debt (including its tax advantage),

$$i \gtrless (1 - f_1)r \; . \tag{24}$$

If debt is cheaper in the relevant scenario, the firm will finance its activities in the final stage with as much debt as possible. If equity is cheaper, which scenario is not purely hypothetical due to the assumption of the capital market being imperfect [see equation (11)], the firm will pay back all its debt before entering the final stage.

Technology

To characterize a scenario it is important to know the relation between the unit costs of both technologies:

$$c_{1XD} \gtrless c_{2XD} \; , \tag{25}$$

$$c_{1XD} = \frac{1}{q_1}\left[w\,l_1 + a + \frac{i}{1-f_1} + \frac{f_2}{1-f_1}e_1\right] ,$$

$$c_{2XD} = \frac{1}{q_2}\left[w\,l_2 + (1 - \frac{g}{1-f_1})a + (1-g)\frac{i}{1-f_1} + \frac{f_2}{1-f_1}e_2\right] .$$

We now discuss the above formulations of c_{1XD} and c_{2XD} in more detail. Concerning c_{1XD}, the part between brackets is the cost per capital good assigned to activity 1, when this capital good is financed by equity only. It is divided by the output per capital good, q_1, in order to obtain the unit cost of activity 1. The cost per capital good consists of four parts:

wages: $w\,l_1$;
depreciation: a;
financing costs: $i/(1 - f_1)$;
cost of pollution: $f_2 e_1/(1 - f_1)$.

The components that contain the costs of wages and depreciation are obvious, so they do not need any further explanation. The financing costs are equal to the desired marginal rate of return to equity before paying profit tax. About the cost of pollution component we can argue that e_1 is equal to the amount of emissions per capital good. The emission is taxed with rate f_2, but it is not allowed to subtract this tax payment from the firm's profit before paying profit tax. Therefore, the tax payment due to the emission per capital good assigned to activity 1 $(f_2 e_1)$ has to be multiplied by the factor $1/(1-f_1)$.

Comparing the unit cost formula of c_{2XD} to that of c_{1XD}, we conclude that the composition of the components that contain the depreciation and financing costs differ. This is caused by the fact that the government offers an investment grant rate g for investments in capital goods assigned to the cleaner technology 2. Hence, a part g of these capital goods is in fact paid by the government. Because no extra profit tax need to be paid upon receiving these investment grants, the costs of depreciation and financing decrease with the amount $(i+a)g/(1-f_1)$ compared to the expression for c_{1XD}.

Now due to the fact that also e_2 is less than e_1, the reader might think that, if l_1 is approximately the same as l_2, c_{1XD} will always be greater than c_{2XD}. However this does not need to be the case, because the higher cost per capital good of activity 1 can be compensated through the higher output per capital good of activity 1, i.e. $q_1 > q_2$. In fact, it depends on the proportion of the values of c_{1XD} and c_{2XD} whether it is more profitable, in the final stage, still to produce by means of the old, less clean production technology 1 or to switch before that stage to production technology 2.

Environmental Policy

The impact of the governmental policy on the pollution of the firm in the final stage of its development is described in the optimality conditions through the next inequality:

$$c_3 \gtrless \frac{f_2}{1-f_1} e_3 \ , \tag{26}$$

in which:

$$c_3 = w\, l_3 + (1 - \frac{g}{1-f_1})a + (1-g)\frac{i}{1-f_1} \ .$$

The left part of (26) are the costs per dollar invested in the cleaning technology 3. Given the technological possibilities, government may decrease these cleaning costs by raising the investment grant rate g. The right part of (26) is the decrease in environmental tax due to lower pollution of e_3 per dollar invested in technology 3. In a scenario with a government stressing on environmental features such as a high investment grant rate g and/or a high emission tax rate f_2, the $<$ sign may hold for (26). In that case, it is worthwhile for the firm to install a cleaning technology in the final stage of its development.

As stated in the beginning of this section, the signs of (24), (25), and (26) fix the final stage towards which will lead the optimal policy of the firm. Different stages of growth and consolidation may precede this final stage. In the next subsections we describe two patterns towards two different final stages. In that way we are able to demonstrate some more interesting features of the optimal solution. In subsection 4.3 the total set of the firm's optimal trajectories is presented.

4.1 The Firm's Optimal Policy Under a Weak Environmental Policy of the Government

Here we analyze a scenario for which the following conditions hold:

$$\text{financing costs:} \quad i < (1 - f_1)r \; ; \tag{27}$$

$$\text{technology:} \quad c_{1XD} < c_{2XD} \; ; \tag{28}$$

$$\text{environmental policy:} \quad c_3 > \frac{f_2}{1 - f_1} e_3 \; . \tag{29}$$

The firm's optimal policy to be studied in this subsection is depicted in Figure 1. The figure shows that the firm starts with maximum borrowing, i.e. $Y = k\,K$, in spite of the fact that debt is the expensive way of financing. This is optimal, because in the beginning marginal

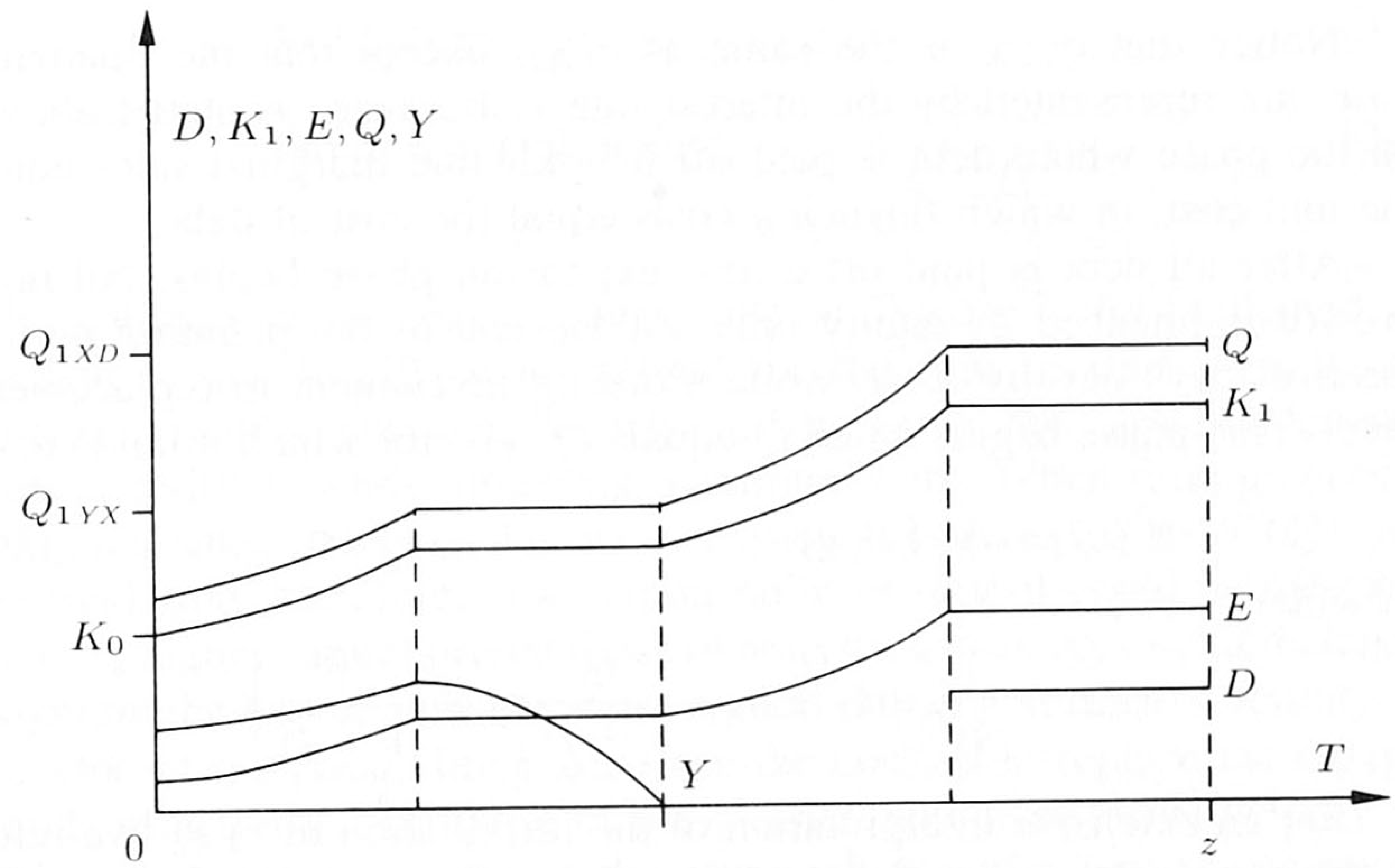

Fig. 1. The firm's optimal policy when debt money is expensive ($i < (1 - f_1)r$) and the government's environmental measures are weak.

sales have such a high value [cf. (22)] that they exceed the marginal cost of an additional capital good, even if this capital good is completely financed by debt money.

As time proceeds marginal sales decrease, because the firm grows and sales are a concave function of the production rate. Now, the firm keeps on financing with maximum debt as long as an additional unit of debt money yields a positive income, i.e. as long as marginal sales exceed the unit cost, in which the financing costs correspond to the case of total debt financing.

At the moment that marginal sales become equal to that unit cost, in which financing costs equal the cost of debt, the firm starts to pay off debt while keeping investments at replacement level. If instead the firm would grow further, then, due to the concavity of the sales function, marginal sales would fall below marginal cost and therefore it is optimal to pay off debt first before growing any further. As soon as this phase of paying off debt starts, debt falls below its upper bound, i.e. $Y < k\,K$, so we arrive at an intermediate debt financing case, which implies that we denote the corresponding unit cost by $c_{1\,YX}$ [cf. (19)]. Hence, on the phase where debt is paid off it holds that:

$$S'(Q) = c_{1\,YX} \;, \tag{30}$$

in which:

$$c_{1\,YX} = \frac{1}{q_1}\left[w\,l_1 + a + \frac{f_2}{1-f_1}e_1 + r\right] .$$

Notice that $c_{1\,YX}$ is the same as $c_{1\,XD}$ except that the financing costs are represented by the interest rate r, because, as stated above, on the phase where debt is paid off it holds that marginal sales equal the unit cost, in which financing costs equal the cost of debt.

After all debt is paid off a new expansion phase begins, but now growth is financed by equity only. At the end of the planning period the firm pays out dividend, while reducing investment to replacement level. This phase begins when Q equals $Q_{1\,XD}$, for which it holds that:

$$S'(Q_{1\,XD}) = c_{1\,XD} \;, \tag{31}$$

in which:

$$c_{1\,XD} = \frac{1}{q_1}\left[w\,l_1 + a + \frac{f_2}{1-f_1}e_1 + \frac{i}{1-f_1}\right] .$$

For an extensive interpretation of the formulation of $c_{1\,XD}$ we refer to the introductory part of this section. From (31) we can conclude that the firm starts paying out dividend when the marginal rate of return to equity exactly equals its desired rate. On the expansion path before this dividend path the marginal rate of return to equity is higher than $i/(1-f_1)$ and therefore it is optimal for the firm to grow at its maximum on this phase.

It is clear that this solution can only occur if: $c_{1XD} < c_{1YX}$, and it is not difficult to derive that this inequality equals the financing costs condition (27).

Another striking characteristic is that during the whole planning period the firm keeps on producing by using the most dirty activity. Obviously, the government's environmental instruments, i.e. the emission tax rate f_2 and the investment grant rate g on cleaner investments, are not sufficiently strong that it is optimal for the firm to exchange a part of its growth for producing output by using cleaner production activities. This is confirmed by the environmental policy condition (29) and also by the technology condition (28).

4.2 The Firm's Optimal Policy Under Strong Environmental Measures of the Government

In the scenario to be studied in this subsection the following conditions are satisfied:

$$\text{financing costs:} \quad i < (1 - f_1)r \; ; \tag{32}$$

$$\text{technology:} \quad c_{1XD} > c_{2XD} \; ; \tag{33}$$

$$\text{environmental policy:} \quad c_3 < \frac{f_2}{1 - f_1} e_3 \; . \tag{34}$$

The solution in this case is presented in Figure 2. As explained in section 3, due to (22) it is optimal for the firm to start growing as fast as possible. This can be achieved by using the capital-extensive dirty activity 1, while attracting maximum debt. When time proceeds, marginal sales decrease due to concavity [Q increases so $S'(Q)$ decreases], and, therefore, at a certain point of time it could be the case that the higher capital productivity of activity 1 does not counterbalance anymore the higher costs per capital good due to pollution of activity 1.

One of the possibilities to reduce the costs is to replace the capital goods of activity 1 by those of the cleaner capital-intensive activity 2. This will happen as soon as the marginal rate of return to equity of activity 1 becomes equal to the marginal rate of return to equity of activity 2. The expression of the marginal rate of return to equity of

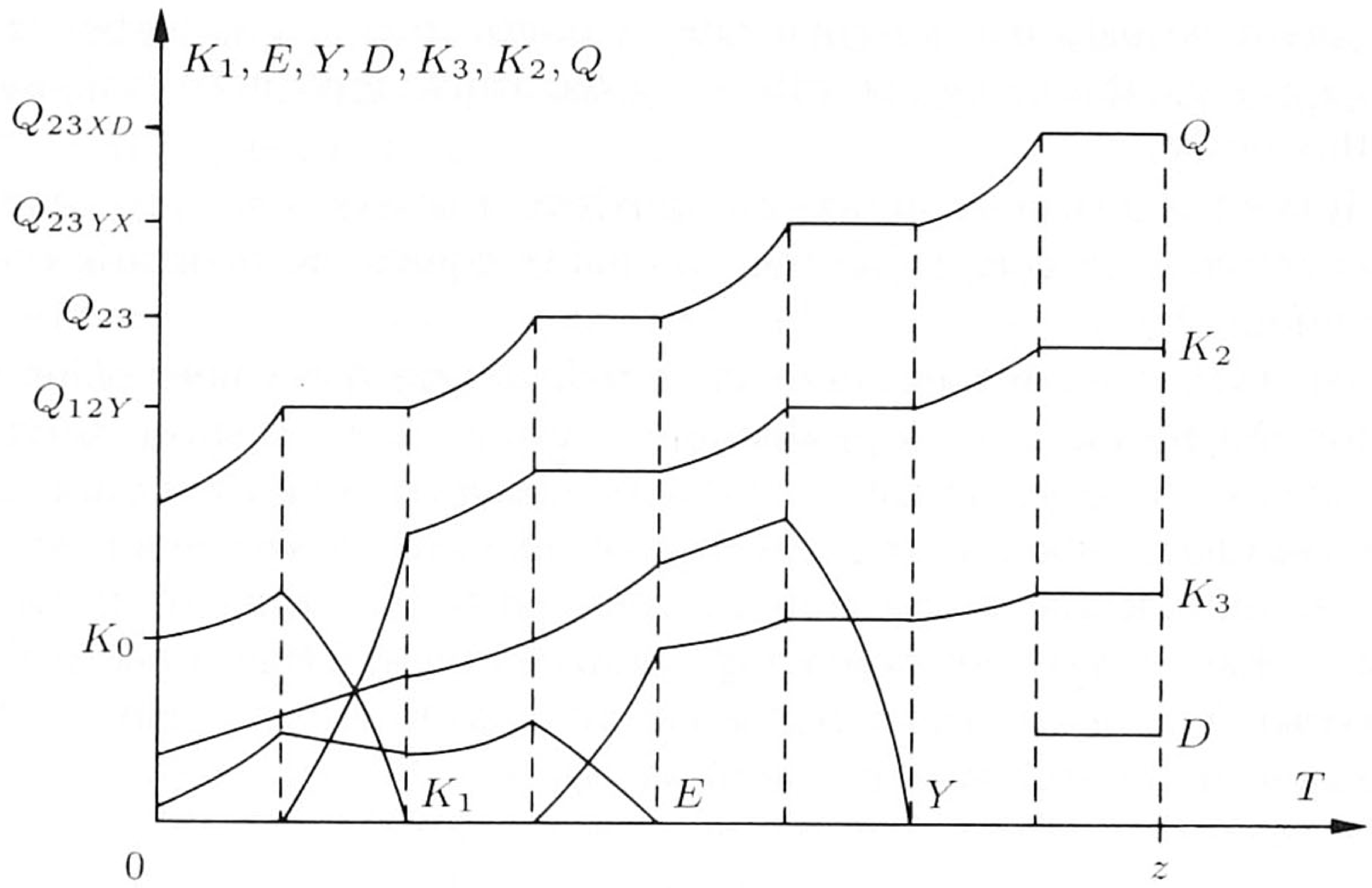

Fig. 2. The firm's optimal policy when debt money is expensive ($i < (1 - f_1)r$) and the government's environmental measures are strong.

activity 1 under maximum debt financing is:

$$\frac{1}{1-k}\left[q_1 S'(Q) - w\,l_1 - a - \frac{f_2}{1-f_1}e_1 - k\,r\right] . \qquad (35)$$

Within brackets we have the marginal rate of return to capital goods. A part of the capital goods is financed by debt, i.e. $Y = k\,K$ [cf. (14)], and therefore the interest cost per capital good equals $k\,r$. To transform the marginal rate of return to capital goods into the marginal rate of return to equity we have to divide the whole thing by $1 - k$, because it holds that $X = (1 - k)K$.

The marginal rate of return to equity of activity 2 under maximum debt financing equals:

$$\frac{1}{1-k-g}\left[q_2 S'(Q) - w\,l_2 - (1 - \frac{g}{1-f_1})a - \frac{f_2}{1-f_1}e_2 - k\,r\right] . \qquad (36)$$

If the firm invests in the cleaner production activity 2, it receives an investment grant g from the government. Between the main brackets of expression (36) depreciation appears net of investment grants. These subsidies may be considered as diminishing the price of capital goods at a rate g, resulting in a decrease of depreciation of $a\,g$ in the case of absence of corporation profit tax. When corporation profit tax is introduced, we have to reckon with the fact that investment grants are free of corporation profit tax, so the relevant decrease of $a\,g$ is then after tax payments and this equals a change of depreciation before taxes of $a\,g/(1-f_1)$. Due to maximum debt financing and the investment grants only $(1-g-k)$ per unit capital is financed by equity, so we have to divide the marginal rate of return to capital goods by $1-g-k$ to obtain the marginal rate of return to equity.

As mentioned before the replacement of the capital goods of activity 1 by those of activity 2 will happen when the marginal rates of return to equity are equal. This holds for $Q = Q_{12Y}$, and this value can be obtained by equalizing (35) and (36):

$$S'(Q_{12Y}) = c_{12Y} \,, \tag{37}$$

in which:

$$c_{12Y} = \frac{1}{(1-k)q_2-(1-k-g)q_1}\Big[\big((1-k)l_2-(1-k-g)l_1\big)w + \frac{k-f_1}{1-f_1}a\,g + g\,k\,r + \big((1-k)e_2-(1-k-g)e_1\big)\frac{f_2}{1-f_1}\Big] \,.$$

After the capital goods of activity 1 have been replaced by those of activity 2, the firm starts growing again but now by using the relatively clean activity 2. When time proceeds marginal sales again decrease and therefore the marginal rate of return to equity will also decrease [cf. (36)]. If the emission tax rate f_2 is relatively high, after some time it may be worthwhile to stop further expansion (and thus more pollution) and to start investing in the non-productive abatement activity 3, while keeping the investment in capital goods of activity 2 at replacement level. Hence, at this stage one unit extra equity will be spend on investing in abatement capital, so the marginal rate of return to equity equals:

$$\frac{1}{1-k-g}\Big[\frac{f_2}{1-f_1}e_3 - w\,l_3 - (1-\frac{g}{1-f_1})a - k\,r\Big] \,. \tag{38}$$

Notice here that one unit extra abatement capital does not lead to extra production, but instead decreases the amount of emission tax to be paid by the firm by $f_2 e_3$, and this must be divided by $1-f_1$, due to the fact that the emission tax cannot be subtracted from profit before paying profit tax.

The investment in the abatement activity starts as soon as the marginal rate of return to equity of activity 2 [cf. (36)] equals the marginal rate of return to equity of activity 3 [cf. (38)]. Hence, the value of Q for which these rates are equal can be obtained by equalizing (36) and (38) and is denoted by Q_{23}:

$$S'(Q_{23}) = c_{23} \ , \tag{39}$$

in which:

$$c_{23} = \frac{1}{q_2}\left[w(l_2 - l_3) + \frac{f_2}{1-f_1}(e_2 + e_3)\right] .$$

Notice that the amount of debt financing does not have any influence on the value of c_{23}, because c_{23} does not contain an interest component. Therefore, the argument that indicates the way of financing is dropped.

After the abatement capacity has reached such a level that all pollution is eliminated, a new expansion phase starts in which a part of the retained earnings is invested in the abatement activity so that the amount of pollution remains zero. The marginal rate of return to equity under maximum debt financing and where the activities 2 and 3 are combined such that there is no pollution, can be expressed as:

$$\begin{aligned}\frac{1}{1-k-g}&\left[q_2 S'(Q)\frac{e_3}{e_2+e_3}\right.\\ &\left.-\left(\frac{l_2 e_3}{e_2+e_3} + \frac{l_3 e_2}{e_2+e_3}\right)w - \left(1 - \frac{g}{1-f_1}\right)a - k\,r\right] .\end{aligned} \tag{40}$$

Due to the absence of pollution, the marginal rate of return to equity does not contain any pollution costs. From (3) we obtain that the elimination of pollution implies that $e_2 K_2 = e_3 K_3$. Within the main brackets we have the marginal rate of return to capital, which implies that this is the extra profit that arises due to the application of an additional capital good. From this capital good $e_3/(e_2+e_3)$ is assigned to activity 2 and $e_2/(e_2+e_3)$ to activity 3, which can be obtained from the two equations $\Delta K_2 + \Delta K_3 = 1$ and $e_2 \Delta K_2 = e_3 \Delta K_3$, implying that $\Delta K_2 = e_3/(e_2+e_3)$ and $\Delta K_3 = e_2/(e_2+e_3)$.

The continued expansion leads to a further decrease of the marginal sales. Therefore, after a while it will be optimal for the firm to reduce its costs by paying off the expensive debt [cf. (32)]. This will happen as soon as the marginal rate of return to equity [cf. (40)] equals the interest rate on debt:

$$\begin{aligned}\frac{1}{1-k-g}&\left[q_2 S'(Q)\frac{e_3}{e_2+e_3} - \left(\frac{l_2 e_3}{e_2+e_3} + \frac{l_3 e_2}{e_2+e_3}\right)w\right.\\ &\left.- \left(1 - \frac{g}{1-f_1}\right)a - k\,r\right] = r .\end{aligned} \tag{41}$$

Growing any further, while still using maximum debt financing, would result in a fall of the marginal rate or return to equity below r. This implies that it is better for the firm to use the marginal dollar for paying off debt than for expansion investments. Therefore it is optimal to pay off debt first before growing any further. If we write Q_{23YX} for Q, expression (41) can be rewritten into:

$$S'(Q_{23YX}) = c_{23YX} , \tag{42}$$

in which:

$$c_{23YX} = \frac{1}{q_2}\left[w\left(l_2 + \frac{e_2}{e_3}l_3\right) + \left((1 - \frac{g}{1-f_1})a + (1-g)r\right)\frac{e_2+e_3}{e_3}\right] .$$

After all debt is paid off, a last expansion phase begins which lasts until the marginal rate of return to equity equals the marginal rate of return desired by the shareholders:

$$\frac{1}{1-g}\left[q_2 S'(Q)\frac{e_3}{e_2+e_3} - (\frac{l_2 e_3}{e_2+e_3} + \frac{l_3 e_2}{e_2+e_3})w - (1 - \frac{g}{1-f_1})a\right] = \frac{i}{1-f_1} . \tag{43}$$

From (43) we can obtain that for the optimal production rate, which we denote by Q_{23XD}, it holds that:

$$S'(Q_{23XD}) = c_{23XD} , \tag{44}$$

in which:

$$c_{23XD} = \frac{1}{q_2}\left[w\left(l_2 + \frac{e_2}{e_3}l_3\right) + \left((1 - \frac{g}{1-f_1})a + (1-g)\frac{i}{1-f_1}\right)\frac{e_2+e_3}{e_3}\right] .$$

During this final stage the retained earnings are used for replacement investment and for paying dividend to the shareholders.

In this subsection we described a situation in which the government's environmental policy is strong enough to force the firm first to replace the capital goods of the dirty activity, and second to eliminate the remaining amount of pollution, still caused by production through the cleaner activity, by investing in a non-productive abatement activity.

The technology condition (33) and the environmental policy condition (34) indicate that it is possible for such a solution to be optimal.

However, to avoid any confusion we repeat that the conditions (32), (33), and (34) are only useful to determine the optimal policy in the final interval. They do not provide any information about the way this final interval is reached. To state this differently, the final policy of investing in activities 2 and 3, only financed by equity, can be preceded through several patterns of intermediate stages. This is shown explicitly in Figure 3 of the next subsection.

4.3 The Total Set of Optimal Trajectories of the Firm

The optimal trajectory of the firm depends on the values of the parameters such as the tax rates, investment grant rate, the labor to capital rates, etc. Each set of parameter values fixes a ranking of the unit costs. In Figure 3 it is shown in what way such rankings correspond to the firm's optimal trajectories.

Due to (22) the firm starts in each trajectory with growing at its maximum by using activity 1 and maximum debt financing. In Figure 3 this feature is pointed out by stating "$1Y$" in the upper square. The optimal policy in the next phase depends on the relationship between the unit costs c_{1YD}, c_{1YX}, c_{12Y}, and c_{13Y}. This is pointed out by stating "max(c_{1YD}, c_{1YX}, c_{12Y}, c_{13Y})" in the diamond below the upper square (see Figure 3). If c_{1YD} has the maximum value of these four unit costs it is optimal for the firm to pay out dividend, while keeping investment at replacement level, as soon as the production rate is such that it holds that:

$$S'(Q) = c_{1YD} \,. \tag{45}$$

If, instead of paying out dividend when the production rate satisfies (45), the firm would go on with expansion investment, the marginal rate of return to equity then falls below the rate desired by the shareholders, so this is not optimal.

In a similar way we can argue that after a while it is optimal to pay off debt if c_{1YX} has the largest value, to replace the capital goods of activity 1 by those of activity 2 if c_{12Y} has the largest value, and to start investing in the non-productive abatement activity 3 if c_{13Y} has the largest value. Now, it is not difficult anymore for the reader to interpret the rest of this figure by himself. The trajectories treated in the subsections 4.1 and 4.2 are pointed out by the solid lines. From "the bottom of the tree" it can be derived that there are sixteen different optimal trajectories, each of which ends with a phase where the firm

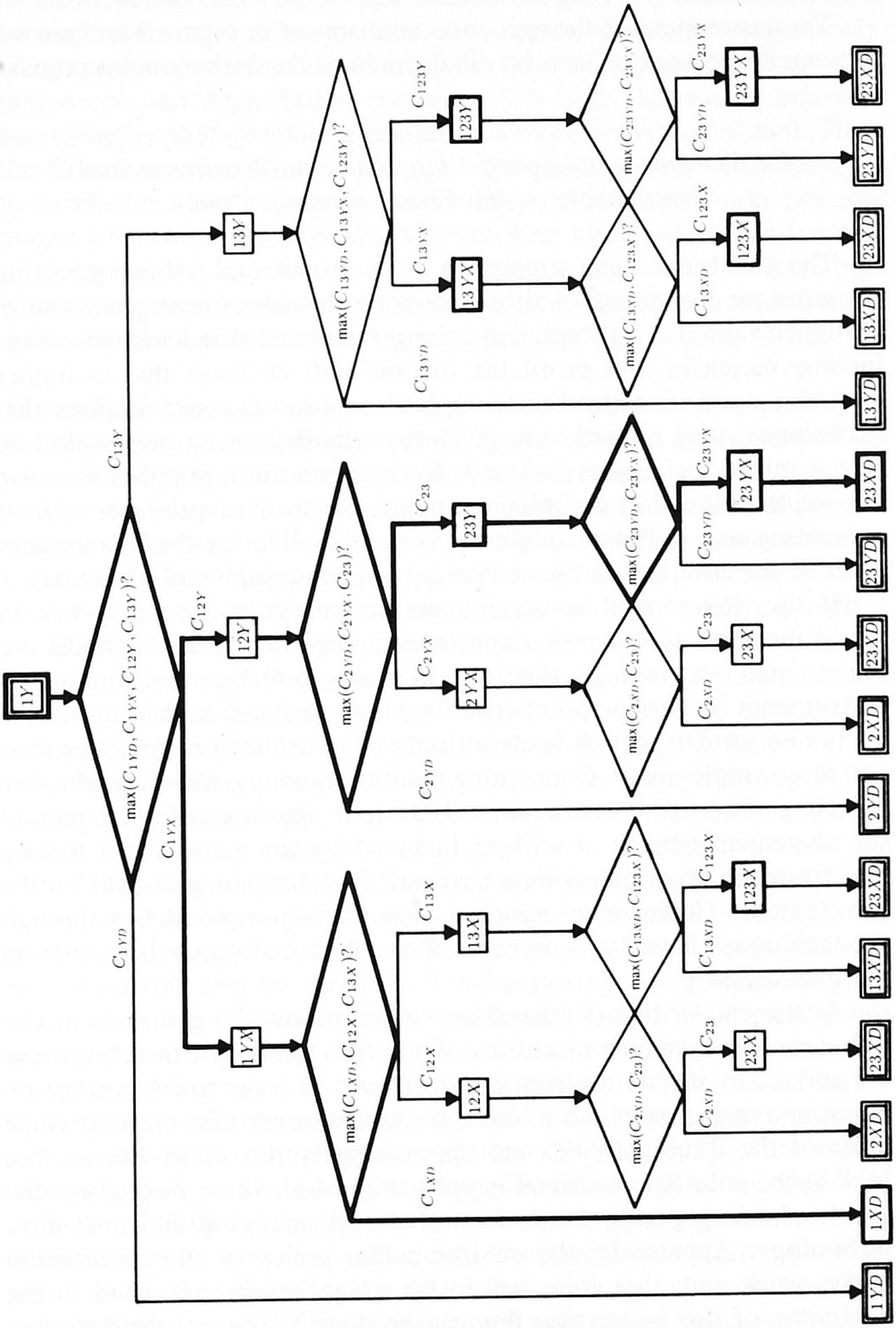

Fig. 3. The firm's optimal trajectories depending on the unit costs.

pays dividend. Of course it must be assumed here that the planning period is sufficiently long so that the final phases can be reached.

The expressions of the unit costs, that appear in Figure 3 and are not presented in the paper, can be obtained from the authors upon request.

4.4 The Impact of Specific Governmental Environmental Instruments on the Firm's Optimal Policy

The government can strengthen its environmental policy by raising the emission tax rate f_2 and/or by raising the investment grant rate g. A higher value for f_2 implies for the government that its emission tax income increases, but profit tax income will decrease due to higher emission costs. For the firm a higher emission tax rate implies that it becomes more attractive to produce without causing any pollution, so the mixed techniques 1–3 and 2–3 become more popular. Because the latter production techniques enable the firm to produce without generating any pollution output, it is not possible for the government to drive the firm out of the market by imposing high emission taxes.

If the government raises the investment grant rate g it has to spend more on subsidizing clean investments, but positive effects are that in this way the government decreases pollution by stimulating investments in cleaner production technologies and it also stimulates economic growth, which leads to higher future profit taxation income and more employment. Concerning the firm, raising g makes production by using the mixed technique 1–3 cheaper, because a bigger part of the abatement activity 3 will be financed by the government (notice that from the investments in activities 2 and 3 a part g is paid by the government). However, a lot more cheaper becomes production through the techniques 2 and 2–3, because then all capital goods benefit from the increase of g.

At the end of this (sub)section we study how the government can influence the optimal trajectories of the firm described in subsections 4.1 and 4.2 by its environmental instruments. In order to concentrate on environmental aspects we assume for the moment that the unit wage costs of the three activities are approximately the same. Let us first look at the solution described in subsection 4.1, where throughout the whole planning period the firm produces by applying the most dirty technology. Apparently, the environmental policy of the government is too weak and, therefore, has to be strengthened. As stated in the beginning of this subsection this can be done by raising the emission tax and/or by raising the investment grants.

Increasing investment grants implies that the government finances a bigger part of the firm's investment in the cleaner technology and in the abatement activity. Hence, these kinds of investments especially become more attractive to the firm if capital goods are very costly, which is the case when the sum of the rates of the shareholders' time preference and depreciation (i.e. $i + a$) is high. Increasing emission taxes implies that generating pollution is more heavily punished. Then, the firm is more willing to change from production by technology 1 to production by the cleaner technology 2 when emission per unit output of technology 2 is much lower than emission per unit output of technology 1, i.e. when $e_1/q_1 - e_2/q_2$ is high. Furthermore, high emission taxes encourage the firm to invest in the abatement activity 3 when the abatement to capital rate of activity 3 (i.e. e_3) is high.

To conclude: when the sum of the rates of shareholders' time preference and depreciation is relatively high the most efficient instrument of the government to tempt the firm to produce more clean is offering grants on investments in cleaner technologies and non-productive abatement activities. On the other hand imposing an emission tax is more effective if producing by a cleaner technology leads to far less pollution or if the abatement to capital rate of the abatement activity is relatively high.[1]

Consider now the firm's trajectory presented in subsection 4.2. From an environmental point of view this solution is characterized by the fact that the firm first switches from producing by the dirty technology 1 to producing by the cleaner technology 2. Later on, the firm further reduces its pollution output to zero by investing in the abatement activity. Suppose the government finds the pollution output generated by technology 2 still too high and, therefore, it wants the firm to switch immediately from using technique 1 to using the mixed technique 1–3 where the amount of abatement capital is as high that the production process does not lead to any pollution. How can this environmental aim be reached? Not by raising the investment grant rate g, because if the firm only uses technology 2 all capital goods are subsidized, while if it uses the mixed technique 1–3 only the capital goods assigned to the abatement activity benefit from a higher investment grant. Hence, increasing the investment grant rate raises the attractivity of technology 2 more than the attractivity of the mixed technique 1–3, so in this way the opposite effect will be reached.

Following this reasoning one might think that the government could reach its aim by lowering g. But then the government's environmental policy is weakened, implying that it becomes attractive for the firm to lengthen the initial time period where only investments in the dirty

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[1] These results can be obtained from the inequalities (25) and (26).

technology 1 take place, and this is the last thing the government wants to happen.

Compared to using technology 2, using the mixed technique 1–3 is made more attractive if the government increases the emission tax rate. This is because production by applying the clean technology 2 still generates pollution output, while application of the mixed technique 1–3 leads to a zero pollution output, implying that in the latter case no emission taxes need to be paid.

5. Conclusions

In this paper the optimal policy of a profit maximizing firm is studied for different scenarios, depending on the costs of available production and abatement activities, financing costs, and governmental policy. The governmental instruments consist of a tax rate on emissions and investment grants that reward investments in capital goods by which the production process leads to less pollution. The problem is analyzed by developing a deterministic dynamic model of the firm which is solved by applying standard control theory. The firm's production process is described by activity analysis (e.g. van Loon, 1983; and Takayama, 1985).

As in van Loon (1983) the firm's optimal trajectories consist of different phases. Each growth phase is followed by a stationary phase in which the firm replaces capital goods of one production activity by those of another, the firm pays off debt, or the firm pays out dividend. At the end of the planning period we always have such a stationary phase with the common characteristic that the firm pays out dividend. Further characteristics of the final phase appear to depend on three clusters of cost differences: between capital costs (equity and debt), between production technologies (with different capital/labor/emission characteristics), and between environmental costs (depending on governmental policy).

In each stationary phase the production rate is fixed by an equality between marginal sales and the unit cost. The explicit formulation of such a unit cost shows how its value depends on capital costs, labor costs, financing costs, emission costs and how on their turn these cost categories are influenced by such parameters as labor- and capital productivity, emission characteristics of the technology in use, investment grant rate, and tax rates on profit and emissions. In this way the influence of the mentioned parameters on the transition from one phase to the other and on the final stationary phase can be analyzed. And, because these parameters can be clustered to such managerial topics as finance, technology, pollution, and governmental policy, this model can be used to study the impact of changes in these fields on optimal managerial policies and so on optimal growth patterns and final stationary phases of the firm. In the paper we gave a demonstration of how this can be done by analyzing the impact of specific governmental environmental instruments. It turned out that offering investment grants on cleaner production technologies and/or on abatement activities are most effective with respect to diminishing pollution output, when the sum of the rates of depreciation and shareholders' time preference is relatively high. On the other hand imposing an emission tax is most effective in case that producing by a cleaner technology leads to far less pollution or if the abatement to capital rate of the abatement activity is relatively high.

A topic of future research could be the development of a differential game between the government and a representative firm in order to get a dynamic interaction between both agents. In the model of this paper governmental policy was exogenously determined. In such a differential game the government may maximize a utility function over time, where utility depends on the amount of pollution and the employment capacity of the firm. In this way the pollution tax rate and investment grant rate can be determined endogenously. A similar kind of research was carried out by Gradus (1989), who studied the influence of the government's profit taxation policy on the optimal dynamic firm behavior within the framework of a differential game.

Appendix 1.
Conditions to Ensure that Activity 1 Produces the Largest Output per Unit Equity

The assumption that under all circumstances the output per unit equity of activity 1 is larger than that of activity 2 is ensured by the following two inequalities:

$$\begin{aligned} &\frac{e_3}{(1-g)e_1+e_3}q_1 > \frac{e_3}{(1-g)(e_1+e_3)}q_2\,; \\ &\frac{e_3}{(1-k-g)e_1+(1-k)e_3}q_1 > \frac{e_3}{(1-k-g)(e_2+e_3)}q_2\,. \end{aligned} \tag{A.1}$$

The first inequality of (A.1) implies that productivity per unit equity of activity 1 combined with activity 3 exceeds the productivity per unit equity of activity 2 combined with activity 3 in the case of no pollution and zero debt financing. To see this consider first the left hand side of this inequality. If K_1 and K_3 are financed by one unit of equity it holds that $K_1 + (1-g)K_3 = 1$ because gK_3 is paid by the government as investment grants. No pollution in case of a combination between the activities 1 and 3 requires that $e_1 K_1 = e_3 K_3$ [cf. (3)]. From these two equalities we obtain that K_1 equals $\frac{e_3}{(1-g)e_1+e_3}$ and this amount of K_1 is able to produce $\frac{e_3}{(1-g)e_1+e_3} q_1$ [cf. (1)]. A similar reasoning can be applied to obtain the expression of the right hand side of the first inequality of (23).

The second inequality of (A.1) has the same meaning as the first one, but it concerns the case of no pollution and maximum debt financing. Due to (4) it is easy to derive that the conditions in (A.1) imply that productivity per unit equity of activity 1 is greater than productivity per unit equity of activity 2 in the self-financing case and no cleaning activities (i.e. $q_1 > \frac{q_2}{1-g}$) as well as in the case of maximum debt financing, where no capital goods are assigned to the abatement activity ($\frac{q_1}{1-k} > \frac{q_2}{1-k-g}$).

Appendix 2.
Reconstruction of the Model

We introduce the following new variables:

$$\bar{K} := K_1 + (1-g)(K_2 + K_3) , \tag{A.2}$$

$$\bar{I} := I_1 + (1-g)(I_2 + I_3) , \tag{A.3}$$

$$C := (1-f_1)\Big[S - \sum_{j=1}^{3} w\, l_j K_j - rY\Big] + f_1 \sum_{i=1}^{3} a K_i - f_2 E , \tag{A.4}$$

in which:

$\bar{K}$ the value of the capital goods stock financed by the firm's own funds;

$\bar{I}$ rate of investment financed by the firm's own funds;

C cash flow after interest and taxes.

After substitution of the above variables in the model, given by equations (1) through (18) in section 3, we can obtain the following simplified model:

$$\underset{K_2,K_3,D,\bar{I}}{\text{maximize}} \int_0^z e^{-iT} D(T)\,\mathrm{d}T + e^{-iz}[\bar{K}(z) - Y(z)] \tag{A.5}$$

subject to

$$\dot{\bar{K}} = \bar{I} - a\bar{K} \,, \tag{A.6}$$

$$\dot{Y} = \bar{I} + D - C \,, \tag{A.7}$$

$$e_1\bar{K} + [e_2 - (1-g)e_1]K_2 - [e_3 + (1-g)e_1]K_3 \geq 0 \,, \tag{A.8}$$

$$k(\bar{K} + g\,K_2 + g\,K_3) - Y \geq 0 \,, \tag{A.9}$$

$$\bar{K} - (1-g)\,(K_2 + K_3) \geq 0 \,, \tag{A.10}$$

$$K_2 \geq 0; \quad K_3 \geq 0; \quad D \geq 0 \,, \tag{A.11}$$

$$Y \geq 0 \,, \tag{A.12}$$

$$\bar{K}(0) - Y(0) = X_0 > 0; \quad \bar{K}(0) = K_1(0) = K_0 > 0 \,, \tag{A.13}$$

in which:

$$Q(\bar{K}, K_2, K_3) = q_1\bar{K} + [q_2 - (1-g)q_1]K_2 - q_1(1-g)K_3 \,, \tag{A.14}$$

$$L(\bar{K}, K_2, K_3) = l_1\bar{K} + [l_2 - l_1(1-g)]K_2 + [l_3 - l_1(1-g)]K_3 \,, \tag{A.15}$$

$$E(\bar{K}, K_2, K_3) = e_1\bar{K} + [e_2 - (1-g)e_1]K_2 - [e_3 + (1-g)e_1]K_3 \,, \tag{A.16}$$

$$\begin{aligned} C(\bar{K}, K_2, K_3, Y) = {} & (1-f_1)\Big[S - w\,l_1\bar{K} - w[l_2 - (1-g)l_1]K_2 \\ & - w[l_3 - (1-g)l_1]K_3 - r\,Y\Big] \\ & + f_1 a[\bar{K} + g\,K_2 + g\,K_3] - f_2 E \,, \end{aligned} \tag{A.17}$$

$$\frac{e_3}{(1-g)e_1+e_3}q_1 > \frac{e_3}{(1-g)\,(e_2+e_3)}q_2 \; ;$$

$$\frac{e_3}{(1-k-g)e_1+(1-k)e_3}q_1 > \frac{e_3}{(1-k-g)\,(e_2+e_3)}q_2 \; , \tag{A.18}$$

$$S = P(Q)Q, \quad S'(Q) \geq 0, \quad S''(Q) < 0 \; , \tag{A.19}$$

a, f_1, f_2, g, i, k, r :
constants with values between zero and one , (A.20)

e_j, l_j, q_j, w, z :
constants which are greater than zero . (A.21)

The simplified model contains two state variables, $\bar{K}$ and Y, four control variables, K_2, K_3, D and $\bar{I}$, one pure state constraint, and six constraints that each contain at least one control variable. Finally, we have two initial conditions represented by (A.13).

Appendix 3. Solution Procedure

We can derive the necessary conditions for an optimal solution by using Pontryagin's Maximum Principle. After applying the direct adjoining approach (see e.g. Feichtinger and Hartl, 1986) the Lagrangian becomes:

$$\begin{aligned} L = e^{-iT}D &+ \psi_1(\bar{I} - a\bar{K}) + \psi_2(\bar{I} + D - C) \\ &+ \lambda_1(e_1\bar{K} + [e_2 - (1-g)e_1]K_2 - [e_3 + (1-g)e_1]K_3) \\ &+ \lambda_2(k[\bar{K} + g\,K_2 + g\,K_3] - Y) \\ &+ \lambda_3(\bar{K} - (1-g)\,(K_2 + K_3)) \\ &+ \lambda_4 K_2 + \lambda_5 K_3 + \lambda_6 D + \lambda_7 Y \; , \end{aligned} \tag{A.22}$$

in which:

ψ_i co-state variable belonging to the i-th state variable, which is continuously differentiable;

λ_j dynamic Lagrange multiplier belonging to the j-th restriction, which is piecewise continuous.

From Corollary 6.3b of Feichtinger and Hartl (1986) it can be derived that the co-state variables really are continuous, because due to the properties of the paths treated later on it will turn out that entry to/exit from a boundary arc of the state constraint always occurs in a non-tangential way.

After some rearranging, the Lagrangian leads to the following conditions, where we remark that (A.23), (A.24), (A.27), and (A.28) can be derived by differentiating the Lagrangian to the control variables $\bar{I}$, D, K_2, and K_3, respectively, and (A.25) and (A.26) arise from equating $-\dot{\psi}_1$ and $-\dot{\psi}_2$ to the partial derivatives of the Lagrangian with respect to the state variables $\bar{K}$ and Y, respectively:

$$\psi_1 + \psi_2 = 0\ , \tag{A.23}$$

$$e^{-iT} + \psi_2 + \lambda_6 = 0\ , \tag{A.24}$$

$$\begin{aligned}\dot{\psi}_1 = -(e^{-iT} + \lambda_6)\,(1 - f_1)\Big[q_1 S'(Q) - w\,l_1 \\ - a - \frac{f_2}{1 - f_1} e_1\Big] - \lambda_1 e_1 - \lambda_2 k - \lambda_3\ ,\end{aligned} \tag{A.25}$$

$$\dot{\psi}_2 = (e^{-iT} + \lambda_6)\,(1 - f_1) r + \lambda_2 - \lambda_7\ , \tag{A.26}$$

$$\begin{aligned}(e^{-iT} + \lambda_6)\,(1 - f_1)\Big[(q_2 - (1 - g) q_1) S'(Q) \\ - w(l_2 - (1 - g) l_1) + \frac{f_1 a\, g}{1 - f_1} \\ - \frac{f_2(e_2 - (1 - g) e_1)}{1 - f_1}\Big] = \\ - \lambda_1(e_2 - (1 - g) e_1) - \lambda_2 k\, g + \lambda_3 (1 - g) - \lambda_4\ ,\end{aligned} \tag{A.27}$$

$$\begin{aligned}(e^{-iT} + \lambda_6)\,(1 - f_1)\Big[-q_1 (1 - g) S'(Q) \\ - w(l_3 - (1 - g) l_1) + \frac{f_1 a\, g}{1 - f_1} \\ + \frac{f_2(e_3 + (1 - g) e_1)}{1 - f_1}\Big] = \\ \lambda_1(e_3 + (1 - g) e_1) - \lambda_2 g\, k + \lambda_3 (1 - g) - \lambda_5\ ,\end{aligned} \tag{A.28}$$

$$\begin{aligned}\lambda_1 \geq 0, \quad \lambda_1(e_1 \bar{K} + (e_2 - (1 - g) e_1) K_2 \\ - (e_3 + (1 - g) e_1) K_3) = 0\ ,\end{aligned} \tag{A.29}$$

$$\lambda_2 \geq 0, \quad \lambda_2(k[\bar{K} + gK_2 + gK_3] - Y) = 0 , \tag{A.30}$$

$$\lambda_3 \geq 0, \quad \lambda_3(\bar{K} - (1-g)(K_2 + K_3)) = 0 , \tag{A.31}$$

$$\lambda_4 \geq 0, \quad \lambda_4 K_2 = 0 , \tag{A.32}$$

$$\lambda_5 \geq 0, \quad \lambda_5 K_3 = 0 , \tag{A.33}$$

$$\lambda_6 \geq 0, \quad \lambda_6 D = 0 , \tag{A.34}$$

$$\lambda_7 \geq 0, \quad \lambda_7 Y = 0 , \tag{A.35}$$

$$\psi_1(z) = e^{-iz} , \tag{A.36}$$

$$\psi_2(z) = -e^{-iz} . \tag{A.37}$$

We can transform the conditions into the optimal trajectories of the firm by applying the "iterative path connecting"-procedure designed by van Loon (1983). The procedure starts with determining the feasible paths. Based on the fact that the Lagrange multipliers λ_j ($j = 1, \ldots, 7$) can be positive or zero, each path is characterized by a combination of positive λ's. However, some of these combinations are infeasible, e.g. λ_2 and λ_7 cannot be positive at the same time, for this would imply that the value of Y equals its upper- und lower-bound at the same time [cf. (A.30) and (A.35)] and this is not possible. In Table 1 we present the feasible paths and their economic features. The mathematical derivations of these features and the expressions of the c's are available from the authors upon request.

To find the optimal trajectories, we start at the horizon date z, and work backwards in time. Hence, we first select those paths that may be final paths. From substitution of (A.36) and (A.37) into (A.23) and (A.24) we obtain that $\lambda_6 = 0$ at the end of the planning period. From this we derive that the paths 4, 5, 9, 10, 18, 19, 25, and 26 may be a final path (cf. Table 1).

Next, we have to start the coupling procedure to construct the optimal trajectories. To see if two paths can be coupled we test whether the following conditions hold:

- continuity of the state variables $\bar{K}$ and Y;
- continuity of the co-state variables ψ_1 and ψ_2;
- continuity of the stock of equity X.

The coupling procedure starts by selecting paths which can precede the final path and proceeds backwards in time. It stops when the set of feasible paths is empty.

Finally, we check if the sequence of paths satisfies the initial

Table 1. The feasible paths.

path	λ_1	λ_2	λ_3	λ_4	λ_5	λ_6	λ_7	K_1	K_2	K_3	E	Y	D	Q[a]
1	0	0	0	+	+	+	+	+	0	0	+	0	0	
2	0	+	0	+	+	+	0	+	0	0	+	kK	0	
3	0	0	0	+	+	+	0	+	0	0	+	+	0	$1YX$
4	0	0	0	+	+	0	+	+	0	0	+	0	+	$1XD$
5	0	+	0	+	+	0	0	+	0	0	+	kK	+	$1YD$
6	0	0	+	0	+	+	+	0	+	0	+	0	0	
7	0	+	+	0	+	+	0	0	+	0	+	kK	0	
8	0	0	+	0	+	+	0	0	+	0	+	+	0	$2YX$
9	0	0	+	0	+	0	+	0	+	0	+	0	+	$2XD$
10	0	+	+	0	+	0	0	0	+	0	+	kK	+	$2YD$
11	0	0	0	0	+	+	+	+	+	0	+	0	0	$12X$
12	0	+	0	0	+	+	0	+	+	0	+	kK	0	$12Y$
13	0	0	0	+	0	+	+	+	0	+	+	0	0	$13X$
14	+	0	0	+	0	+	+	+	0	+	0	0	0	
15	0	+	0	+	0	+	0	+	0	+	+	kK	0	$13Y$
16	+	+	0	+	0	+	0	+	0	+	0	kK	0	
17	+	0	0	+	0	+	0	+	0	+	0	+	0	$13YX$
18	+	0	0	+	0	0	+	+	0	+	0	0	+	$13XD$
19	+	+	0	+	0	0	0	+	0	+	0	kK	+	$13YD$
20	0	0	+	0	0	+	+	0	+	+	+	0	0	23
21	+	0	+	0	0	+	+	0	+	+	0	0	0	
22	0	+	+	0	0	+	0	0	+	+	+	kK	0	23
23	+	+	+	0	0	+	0	0	+	+	0	kK	0	
24	+	0	+	0	0	+	0	0	+	+	0	+	0	$23YX$
25	+	0	+	0	0	0	+	0	+	+	0	0	+	$23XD$
26	+	+	+	0	0	0	0	0	+	+	0	kK	+	$23YD$
27	+	0	0	0	0	+	+	+	+	+	0	0	0	$123X$
28	+	+	0	0	0	+	0	+	+	+	0	kK	0	$123Y$

[a] $1YX$ in the column below Q means: $S'(Q) = c_{1YX}$.

conditions. In this case they consist of (A.13) and the assumption that the firm starts producing by using the capital-extensive dirty activity (see section 2).

Application of the above described procedure leads to sixteen different feasible solutions, some of which are treated in section 4. A survey of the complete solution and its mathematical derivation can again be obtained from the authors upon request.

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